Chap 23: Instrumental Variables Analysis of Randomized Experiments with One-Sided Noncompliance

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Introduction

How to analyze causal effects if the treatment is confounded?

- consider alternatives to the unconfoundedness assumption
- Instrumental Variables

Target: estimate causal effect of assignment trt on outcome for subpopulation of compliers. (Local Average Treatment Effect)

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Terms

- IV: Instrumental Variables
- CO, NC: Compliance, Non-Compliance
- One-sided noncompliance
- ITT analysis: Intention-to-Treat analysis
- Exclusion restriction
- LATE: Local Average Treatment Effect

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Table

Table 23.1. *Sommer–Zeger Vitamin Supplement Data*

Compliance Type	Assignment Z_i	Vitamin Supplements W_i^{obs}	Survival Y_i^{obs}	Number of Units ($N = 23{,}682$)
co or nc	0	0	0	74
co or nc	0	0	1	11,514
nc	1	0	0	34
nc	1	0	1	2385
co	1	1	0	12
co	1	1	1	9663

- Y_i^{obs}: observed outcome (dead / survive)
- $W_i^{obs} \in \{0, 1\}$: treatment of interest
- $Z_i \in \{0, 1\}$: assignment to the treatment

Setup

$$W_i^{obs} = W_i(Z_i) = \begin{cases} W_i(0) & \text{if } Z_i = 0 \\ W_i(1) & \text{if } Z_i = 1 \end{cases}$$

$$Y_i^{obs} = Y_i(Z_i, W_i^{obs}) = \begin{cases} Y_i(0,0), & \text{if } Z_i = 0, W_i^{obs} = 0 \\ Y_i(1,0), & \text{if } Z_i = 1, W_i^{obs} = 0 \\ Y_i(1,1), & \text{if } Z_i = 1, W_i^{obs} = 1 \end{cases}$$

Setup

- Subsample sizes by treatment assignment and treatment received

$$N_0 = \sum_{i=1}^{N}(1 - Z_i), \quad N_1 = \sum_{i=1}^{N} Z_i$$

$$N_c = \sum_{i=1}^{N}(1 - W_i^{obs}), \quad N_t = \sum_{i=1}^{N} W_i^{obs}$$

- sample sizes by both treatment assigned and received

$$N_{0c} = \sum_{i=1}^{N}(1 - Z_i) \cdot (1 - W_i^{obs}), \quad N_{0t} = \sum_{i=1}^{N}(1 - Z_i) \cdot W_i^{obs}$$

$$N_{1c} = \sum_{i=1}^{N} Z_i \cdot (1 - W_i^{obs}), \quad N_{1t} = \sum_{i=1}^{N} Z_i \cdot W_i^{obs}$$

Setup

- Average outcomes and average treatment received by assignment

$$\bar{Y}_0^{obs} = \frac{1}{N_0}\sum_{i=1}^{N}(1 - Z_i) \cdot Y_i^{obs}, \quad \bar{Y}_1^{obs} = \frac{1}{N_1}\sum Z_i \cdot Y_i^{obs}$$

$$\bar{W}_0^{obs} = \frac{1}{N_0}\sum_{i=1}^{N}(1 - Z_i) \cdot W_i^{obs}, \quad \bar{W}_1^{obs} = \frac{1}{N_1}\sum Z_i \cdot W_i^{obs}$$

- Average outcomes by treatment received

$$\bar{Y}_c^{obs} = \frac{1}{N_c}\sum_{i=1}^{N}(1 - W_i^{obs}) \cdot Y_i^{obs}, \quad \bar{Y}_t^{obs} = \frac{1}{N_t}\sum W_i^{obs} \cdot Y_i^{obs}$$

Setup

- Average outcomes by both treatment assignment and received

$$\bar{Y}_{0c}^{obs} = \frac{1}{N_{0c}} \sum_{i=1}^{N} (1 - Z_i) \cdot (1 - W_i^{obs}) \cdot Y_i^{obs},$$

$$\bar{Y}_{0t}^{obs} = \frac{1}{N_{0t}} \sum (1 - Z_i) \cdot W_i^{obs} \cdot Y_i^{obs}$$

$$\bar{Y}_{1c}^{obs} = \frac{1}{N_{1c}} \sum_{i=1}^{N} (Z_i) \cdot (1 - W_i^{obs}) \cdot Y_i^{obs},$$

$$\bar{Y}_{1t}^{obs} = \frac{1}{N_{1t}} \sum Z_i \cdot W_i^{obs} \cdot Y_i^{obs}$$

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Intention To Treat Effects

- Target is to avoid the problem of noncompliance
- By focusing on the relationship between Z_i and Y_i

ITT Estimands

- ITT effect on the receipt of treatment

$$\text{ITT}_W = \frac{1}{N}\sum_{i=1}^{N}(W_i(1) - W_i(0)) = \frac{1}{N}\sum_{i=1}^{N} W_i(1)$$

- unbiased estimator for ITT_W and sample variance

$$\widehat{\text{ITT}_W} = \bar{W}_i^{obs} - \bar{W}_0^{obs} = \bar{W}_1^{obs}$$

$$\hat{\mathbb{V}}(\widehat{\text{ITT}_W}) = \frac{s_{W,0}^2}{N_0} + \frac{S_{W,1}^2}{N_1} = \frac{1}{N_1 - 1} \cdot \bar{W}_1^{obs} \cdot (1 - \bar{W}_1^{obs})$$

ITT Estimands

- ITT effect on Y

$$\text{ITT}_Y = \frac{1}{N}\sum_{i=1}^{N}(Y_i(1, W_i(1)) - Y_i(0, W_i(0)))$$

- unbiased estimator for ITT_Y and sample variance

$$\widehat{\text{ITT}_Y} = \bar{Y}_1^{obs} - \bar{Y}_0^{obs}$$

$$\hat{\mathbb{V}}(\widehat{\text{ITT}_Y}) = \frac{s_{Y,0}^2}{N_0} + \frac{S_{S,1}^2}{N_1}$$

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Compliers and Noncompliers

- Compliance status
  $$G_i = \begin{cases} co & \text{if } W_i(1) = 1 \\ nc & \text{if } W_i(1) = 0 \end{cases}$$
- N_{co}, N_{nc}: number of units of each type in the sample
- π_{co}, π_{nc}: sample fractions of compliers and noncomliers

  For noncompliers, $W_i(z) = 0$ for $z = 0, 1$

ITT Effect by compliance status

Table 23.2. ***Possibly Compliance Status by Observed Assignment and Receipt of Treatment for the Sommer-Zeger Vitamin Supplement Data***

		Assignment Z_i	
		0	1
Receipt of treatment W_i^{obs}	0	nc or co	nc
	1	–	co

Note: One-sided noncompliance rules out the $Z_i = 0$ $W_i^{obs} = 1$ cell.

ITT Effect by compliance status

- ITT Effect on treatment by compliance status

$$\text{ITT}_W = \pi_{nc} \cdot \text{ITT}_{W,nc} + \pi_{co} \cdot \text{ITT}_{W,co} = \pi_{co}$$

where

$$\text{ITT}_{W,nc} = \frac{1}{N_{nc}} \sum_{i:G_i=nc} (W_i(1) - W_i(0)) = 0$$
$$\text{ITT}_{W,co} = \frac{1}{N_{co}} \sum_{i:G_i=co} (W_i(1) - W_i(0)) = 1$$

ITT Effect by compliance status

- ITT Effect on the outcome by compliance status

$$\begin{aligned}\text{ITT}_Y &= \text{ITT}_{Y,co} \cdot \pi_{co} + \text{ITT}_{Y,nc} \cdot \pi_{nc} \\ &= \text{ITT}_{Y,co} \cdot \text{ITT}_W + \text{ITT}_{Y,nc} \cdot (1 - \text{ITT}_W)\end{aligned}$$

where

$$\text{ITT}_{Y,nc} = \frac{1}{N_{nc}} \sum_{i:G_i=nc} (Y_i(1,0) - Y_i(0,0))$$

$$\text{ITT}_{Y,co} = \frac{1}{N_{co}} \sum_{i:G_i=co} (Y_i(1,1) - Y_i(0,0))$$

- two ITT effects on Y cannot be estimated directly from the observable data
- disentangle ITT effects by compliance type under the exclusion restriction assumption
- this is because W_i^{obs} is unconfounded conditional on compliance status

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Exclusion Restriction for Noncompliers

- Exclusion restriction for noncompliers

  - For all noncompliers, $Z_i \perp Y_i(W_i(Z_i))$

  i.e.) super population distribution of $Y_i(0,1)$ is same as $Y_i(1,0)$ for noncompliers.

Local Average Treatment Effects

- Average ITT effect in the population can be decomposed as;

$$\text{ITT}_Y = \text{ITT}_{Y,co} \cdot \text{ITT}_W + \text{ITT}_{Y,nc} \cdot (1 - \text{ITT}_W)$$

- And by usning exclusion restriction for noncompliers ($\text{ITT}_{Y,nc} = 0$),

$$\text{ITT}_Y = \text{ITT}_{Y,co} \cdot \text{ITT}_W$$

- Hence,

$$\tau_{late} = \text{ITT}_{Y,co} = \frac{\text{ITT}_Y}{\text{ITT}_W}$$

- As a result, with exclusion restriction given, LATE can be interpreted as average causal effect of the receipt of treatment for compliers

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Conclusion

- With one-sided noncompliance setting, unconfoundedness assumption is broken
- By assuming unconfoundedness of assignment and exclusion restriction, we can estimate LATE